

ПЪРВА НАЦИОНАЛНА СТУДЕНТСКА ОЛИМПИАДА  
ПО КОМПЮТЪРНА МАТЕМАТИКА  
„АКАДЕМИК СТЕФАН ДОДУНЕКОВ”  
ТЕХНИЧЕСКИ УНИВЕРСИТЕТ - ГАБРОВО  
24-26. X. 2012 г.

**Решения на задачите за група Б**

**Задача 1**

$$x = 5.1$$

$$y = 3.14$$

$$\sqrt[3]{x+y} + \sqrt[3]{x-y}$$

Remove[x, y]

$$5.1$$

$$3.14$$

$$3.27127$$

**Задача 2**

NSolve[x^2 - 5 \* x + 11 == 0, x]

{{x -> 2.5 - 2.17945 i}, {x -> 2.5 + 2.17945 i}}

(2.5 - 2.179449471770337 i)^12 + (2.5 + 2.179449471770337 i)^12

-2.41528 x 10^6 + 0. i

Remove[x]

**Задача 3**

Expand[Collect[(x - 1) (x + 2) (x - 3) (x + 4) (x - 5) (x + 6), x]]

-720 + 444 x + 400 x^2 - 87 x^3 - 41 x^4 + 3 x^5 + x^6

Remove[x]

**Задача 4**

z[x\_] := x^5 + (5 - a) x^4 - (5 a + 7) x^3 + (7 a - 29) x^2 + (29 a + 30) x - 30 a

Expand[z[x]]

-30 a + 30 x + 29 a x - 29 x^2 + 7 a x^2 - 7 x^3 - 5 a x^3 + 5 x^4 - a x^4 + x^5

Изнасяме x-а пред скоби и имаме:

z = (x - a)(x^4 + 5x^3 - 7x^2 - 29x + 30)

Solve[x^4 + 5 x^3 - 7 x^2 - 29 x + 30 == 0, x]

{{x -> -5}, {x -> -3}, {x -> 1}, {x -> 2}}

Следователно за a = -5, -3, 1 или 2 полиномът ще има двукратна нула.

Remove[x, z]

**Задача 5**

`Solve[x^4 - 5 x^2 + 10 x - 6 == 0, x, Complexes]`

`{{x → -3}, {x → 1}, {x → 1 - i}, {x → 1 + i}}`

`Remove[x]`

### Задача 6

`Reduce[x^2 - x - 2 > 0, x]`

`x < -1 || x > 2`

Първи случай:  $e^x < -1$  е невъзможен

Втори случай:

`Reduce[e^x > 2, x, Reals]`

`x > Log[2]`

`Remove[x]`

### Задача 7

`a = {{1, 1, x}, {0, 1, 1}, {0, 0, 1}}`

`b = {{1, 2012, x}, {0, 1, 2012}, {0, 0, 1}}`

`c = MatrixPower[a, 2012]`

`Solve[c == b, x]`

`{{1, 1, x}, {0, 1, 1}, {0, 0, 1}}`

`{{1, 2012, x}, {0, 1, 2012}, {0, 0, 1}}`

`{{1, 2012, 2 023 066 + 2012 x}, {0, 1, 2012}, {0, 0, 1}}`

`{{x → -1006}}`

`Remove[x, a, b, c]`

### Задача 8

`Reduce[`

`{a, b}, {c, d} . {{1, 2, 3}, {4, 5, 6}} == {{18, 24, 30}, {29, 40, 51}}, {a, b, c, d}]`

`a == 2 && b == 4 && c == 5 && d == 6`

Отговор:

`MatrixForm[{{2, 4}, {5, 6}}`

$$\begin{pmatrix} 2 & 4 \\ 5 & 6 \end{pmatrix}$$

`Remove[a, b, c, d, x, y]`

### Задача 9

`RSolve[{a[n] == a[n - 1] - 2 * a[n - 2], a[1] == 1, a[2] == 2}, a[n], n]`

`{{a[n] → -`

$$\frac{1}{14(-i + \sqrt{7})}$$

`(-21 i (`

$$\left( -21 i \left( \frac{1}{2} - \frac{i \sqrt{7}}{2} \right)^n + \sqrt{7} \left( \frac{1}{2} - \frac{i \sqrt{7}}{2} \right)^n + 14 i \left( \frac{1}{2} + \frac{i \sqrt{7}}{2} \right)^n + 6 \sqrt{7} \left( \frac{1}{2} + \frac{i \sqrt{7}}{2} \right)^n \right) \}}$$

$$\begin{aligned}
 a_{102} = & -\frac{1}{14(-i + \sqrt{7})} \left( -21i \left( \frac{1}{2} - \frac{i\sqrt{7}}{2} \right)^n + \right. \\
 & \left. \sqrt{7} \left( \frac{1}{2} - \frac{i\sqrt{7}}{2} \right)^n + 14i \left( \frac{1}{2} + \frac{i\sqrt{7}}{2} \right)^n + 6\sqrt{7} \left( \frac{1}{2} + \frac{i\sqrt{7}}{2} \right)^n \right) /. n \rightarrow 102 \\
 & - \frac{1}{14(-i + \sqrt{7})} \\
 & \left( -21i \left( \frac{1}{2} - \frac{i\sqrt{7}}{2} \right)^{102} + \sqrt{7} \left( \frac{1}{2} - \frac{i\sqrt{7}}{2} \right)^{102} + 14i \left( \frac{1}{2} + \frac{i\sqrt{7}}{2} \right)^{102} + 6\sqrt{7} \left( \frac{1}{2} + \frac{i\sqrt{7}}{2} \right)^{102} \right)
 \end{aligned}$$

$$\begin{aligned}
 a_{101} = & -\frac{1}{14(-i + \sqrt{7})} \left( -21i \left( \frac{1}{2} - \frac{i\sqrt{7}}{2} \right)^n + \right. \\
 & \left. \sqrt{7} \left( \frac{1}{2} - \frac{i\sqrt{7}}{2} \right)^n + 14i \left( \frac{1}{2} + \frac{i\sqrt{7}}{2} \right)^n + 6\sqrt{7} \left( \frac{1}{2} + \frac{i\sqrt{7}}{2} \right)^n \right) /. n \rightarrow 101 \\
 & - \frac{1}{14(-i + \sqrt{7})} \\
 & \left( -21i \left( \frac{1}{2} - \frac{i\sqrt{7}}{2} \right)^{101} + \sqrt{7} \left( \frac{1}{2} - \frac{i\sqrt{7}}{2} \right)^{101} + 14i \left( \frac{1}{2} + \frac{i\sqrt{7}}{2} \right)^{101} + 6\sqrt{7} \left( \frac{1}{2} + \frac{i\sqrt{7}}{2} \right)^{101} \right)
 \end{aligned}$$

```
r = MatrixPower[{{1, -2}, {1, 0}}, 100].{2, 1}
```

```
{-832162467610076, 1286059417395116}
```

```
Reduce[{a102, a101} == r]
```

```
True
```

```
Remove[a, n, r, a101, a102]
```

### Задача 10

```
Solve[{\alpha * x1 + x2 + x3 == 1,
```

```
  x1 + \alpha * x2 + x3 == 1, x1 + x2 + \alpha * x3 == \alpha^2}, {x1, x2, x3}]
```

```
{ {x1 -> -\frac{\alpha}{2 + \alpha}, x2 -> -\frac{\alpha}{2 + \alpha}, x3 -> -\frac{-2 - 2\alpha - \alpha^2}{2 + \alpha}} }
```

```
Remove[\alpha, x1, x2, x3]
```

### Задача 11

Намираме координатите на връх А:

```
Reduce[x + 11 y - 23 == 0 && 10 x - y - 8 == 0, {x, y}]
```

```
x == 1 && y == 2
```

```
a = {1, 2}
```

```
{1, 2}
```

Намираме координатите на връх В:

```
Reduce[x + 11 y - 23 == 0 && 11 x + 10 y - 142 == 0, {x, y}]
```

```
x == 12 && y == 1
```

```
b = {12, 1}
```

```
{12, 1}
```

Намираме координатите на връх C:

```
Reduce[10 x - y - 8 == 0 && 11 x + 10 y - 142 == 0, {x, y}]
```

```
x == 2 && y == 12
```

Символът C е защитен; използваме C1 за връх C:

```
C1 = {2, 12}
```

```
{2, 12}
```

```
ab = b - a
```

```
ac = C1 - a
```

```
{11, -1}
```

```
{1, 10}
```

```
s = 0.5 Abs[Det[{ab, ac}]]
```

```
55.5
```

```
Remove[x, y, a, b, C1, A, B, ab, ac, AB, AC]
```

### Задача 12

```
Solve[{(x - 4)^2 + (y - 3)^2 == 10, (x - 7)^2 + (y - 5)^2 == 9}, {x, y}]
```

```
{{{x -> 55/13, y -> 80/13}, {x -> 7, y -> 2}}
```

```
sreda = 0.5 * ({55/13, 80/13} + {7, 2})
```

```
{5.61538, 4.07692}
```

```
Remove[x, y, sreda]
```

### Задача 13

```

pZY = {0, 3, -2}
pXZ = {1, 0, -2}
pXY = {1, 3, 0}
n = {x, y, z}
Solve[Det[{n - pZY, n - pXZ, n - pXY}] == 0, {x, y, z}]
{0, 3, -2}
{1, 0, -2}
{1, 3, 0}
{x, y, z}

```

Solve::svars: Equations may not give solutions for all "solve" variables. >>

$$\left\{ \left\{ z \rightarrow -4 + 2x + \frac{2y}{3} \right\} \right\}$$

Следователно уравнението на равнината има вида:

$$2x + (2/3)y - z - 4 = 0$$

```
Remove[pZY, pXZ, pXY, x, y, z, n]
```

#### Задача 14

```

a = {1, 2, 4}
b = {5, 1, 3}
c = {4, 2, 1}
d = {2, x, 5}
{1, 2, 4}
{5, 1, 3}
{4, 2, 1}
{2, x, 5}

```

```
Solve[1 / 6 Abs[Det[{a - b, b - c, c - d}]] == 2, x]
```

Solve::ifun: Inverse functions are being used by Solve, so  
some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{ \{x \rightarrow 0\}, \left\{ x \rightarrow \frac{8}{3} \right\} \right\}$$

```
Remove[x, a, b, c, d]
```

#### Задача 15

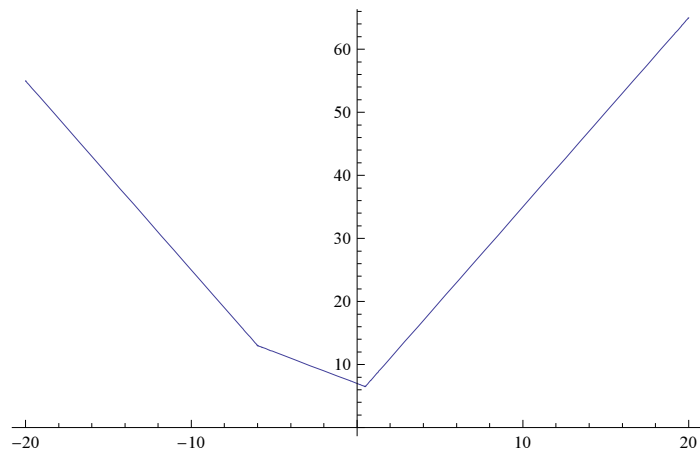
```

Solve[{Abs[1 - a] + Abs[2 + b] == 8, Abs[-3 - a] + Abs[-6 + b] == 10}, {a, b}]
{{a -> -6, b -> -1}, {a -> 4, b -> 3}}

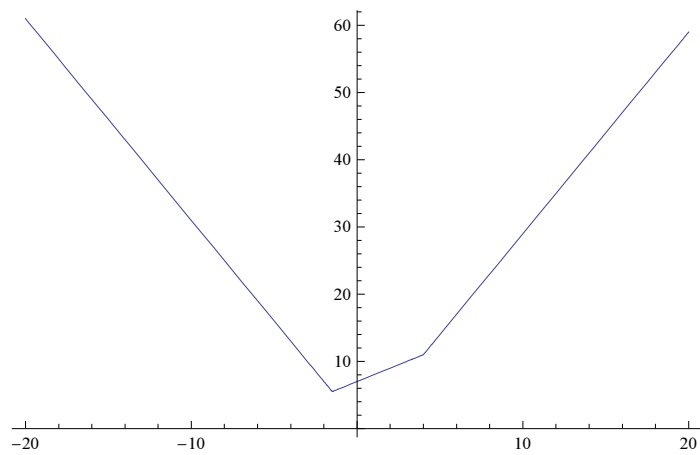
```

1 случай:

```
Plot[Abs[x + 6] + Abs[2 x - 1], {x, -20, 20}]
```



```
Plot[Abs[x - 4] + Abs[2 x + 3], {x, -20, 20}]
```



```
Remove[a, b, x]
```

### Задача 16

```
Limit[(1 + Cot[x]) ^ (Tan[x]), x -> Pi / 2]
```

e

```
Remove[x]
```

### Задача 17

```
f[x_] := Log[Sin[2 * x] + Cos[3 * x]]
```

```
f''[Pi / 6]
```

$$-\frac{28}{3}$$

```
Remove[f, x]
```

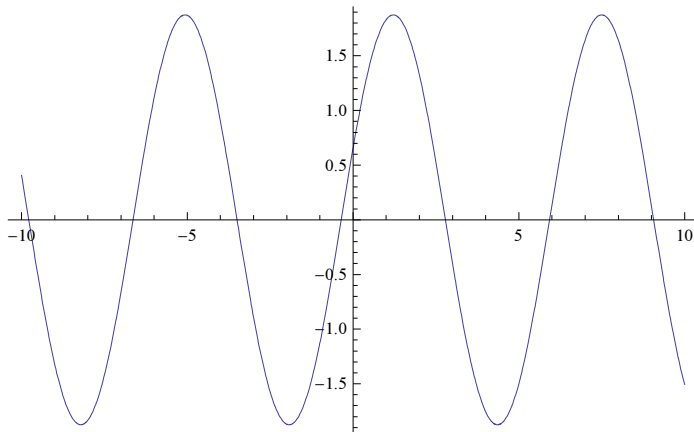
### Задача 18

```
f[x_] := Det[
  {{1, x, x, x, x}, {x, 2, x, x, x}, {x, x, 3, x, x}, {x, x, x, 4, x}, {x, x, x, x, 5}}]
FindMaximum[{f[p], 1 ≤ p ≤ 5}, p]
FindMinimum[{f[q], 1 ≤ q ≤ 5}, q]
{24., {p → 1.}}
{-29.0985, {q → 4.22474}}
```

```
Remove[f, x, p, q]
```

### Задача 19

```
Plot[Sin[x] - Cos[4 - x], {x, -10, 10}]
```



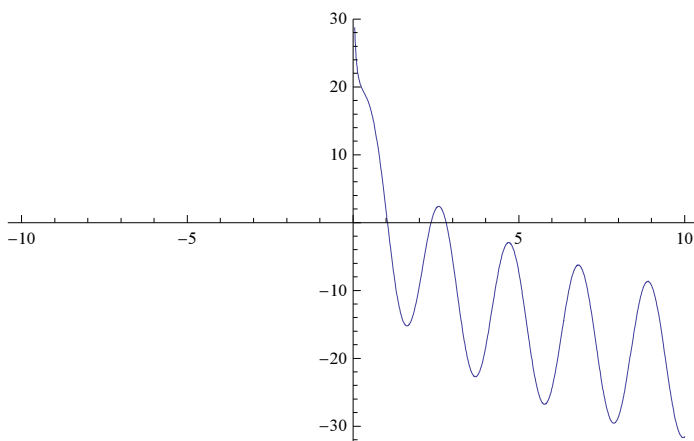
$\text{Sin}[x] - \text{Cos}[x] = 0$  има безброй много решения, а най-малкото положително се намира в интервала  $[0, 5]$

```
NSolve[Sin[x] - Cos[4 - x] == 0 && x ≥ 0 && x ≤ 5, x]
{{x → 2.7854}}
```

```
Remove[x]
```

### Задача 20

```
Plot[11 * Sin[3 * x] - 9 * Log[x], {x, -10, 10}]
```



От графиката се вижда, че уравнението има 3 корена, близо до точките 1, 2.5 и 3

```

FindRoot[11 * Sin[3 * x] == 9 * Log[x], {x, 1}]
FindRoot[11 * Sin[3 * x] == 9 * Log[x], {x, 2.5}]
FindRoot[11 * Sin[3 * x] == 9 * Log[x], {x, 3}]
{x → 1.03723}
{x → 2.3529}
{x → 2.8065}

```

```
Remove[x]
```

### Задача 21

```
z = D[Sin[x] + 3 * a * Sin[2 x] - 1 / 3 Sin[3 x] - 6 a * x, x]
```

```
- 6 a + Cos[x] + 6 a Cos[2 x] - Cos[3 x]
```

```
TrigFactor[z]
```

```
16 Cos[x/2]^2 (-3 a + Cos[x]) Sin[x/2]^2
```

```
Reduce[-3 a + Cos[x] ≥ 0, x, Reals]
```

```
a ≤ -1/3 ||
```

```
(C[1] ∈ Integers && -1/3 < a < 1/3 && -ArcCos[3 a] + 2 π C[1] ≤ x ≤ ArcCos[3 a] + 2 π C[1]) ||
```

```
(C[1] ∈ Integers && a == 1/3 && x == 2 π C[1])
```

```
Remove[z, x, a]
```

### Задача 22

Разглеждаме функцията в интервала  $x \geq 1$  и интегрираме

```
f1 = Integrate[x - 1, x]
```

```
-x + x^2/2
```

Разглеждаме функцията в интервала  $(-\infty; 1]$  и интегрираме

```
f2 = Integrate[1 - x, x]
```

```
x - x^2/2
```

Тогава примитивната функция има вида:

```
F = Piecewise[{{f1, x ≥ 1}, {f2, x ≤ 1}}]
```

$$\begin{cases} -x + \frac{x^2}{2} & x \geq 1 \\ x - \frac{x^2}{2} & x \leq 1 \\ 0 & \text{True} \end{cases}$$

```
Remove[x, f1, F]
```

### Задача 23



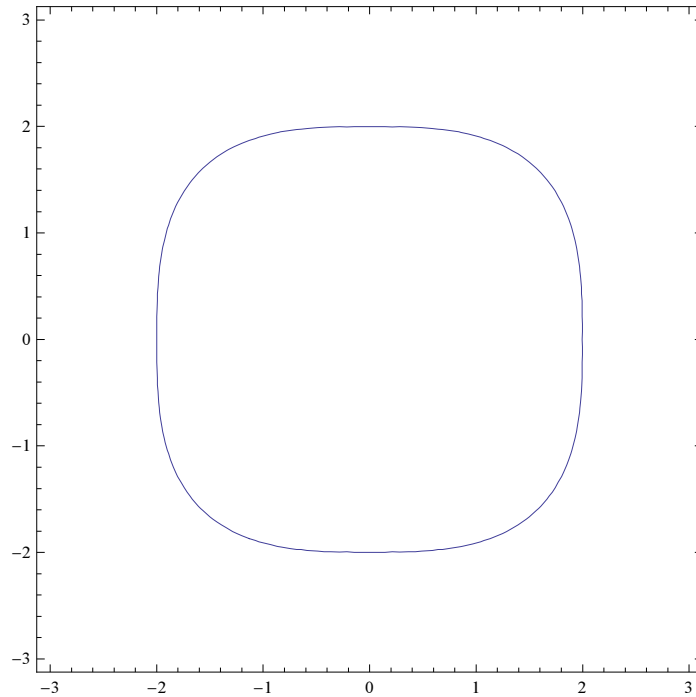
```
Integrate[1 / (1 + Sin[x] + Cos[x]), {x, 0, Pi / 2}]
```

```
Log[2]
```

```
Remove[x]
```

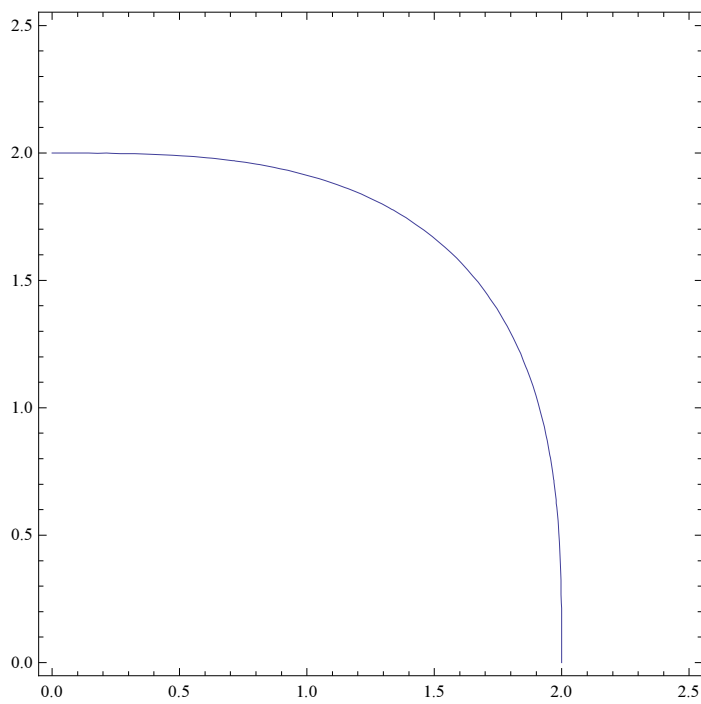
### Задача 24

```
ContourPlot[Abs[x]^3 + Abs[y]^3 == 8, {x, -3, 3}, {y, -3, 3}]
```



Лицето на цялата област е 4 пъти лицето на областта под тази крива:

```
ContourPlot[x^3 + y^3 == 8, {x, 0, 2.5}, {y, 0, 2.5}]
```



```
S = 4 * Integrate[Integrate[x^3 + y^3, {x, 0, 2}], {y, 0, 2}]
```

```
64
```

```
Remove[x, y, S, s1]
```

### Задача 25

```
Solve[Sum[1/(k*(k+1)*(k+2)), {k, 1, n}] == 1013545/4054182 && n >= 0, n, Integers]
```

```
{{n -> 2012}}
```

```
Remove[n, k]
```

### Задача 26

```
Solve[Limit[Sum[Cos[k*x], {k, 1, n}], x -> Pi/3] == 1/2 && n >= 2000 && n <= 2012, n]
```

```
{{n -> 2005}, {n -> 2005}, {n -> 2011}, {n -> 2011}}
```

```
Remove[n, k, x]
```

### Задача 27

```
Reduce[Integrate[(a*t + b*t^2) Cos[m*t], {t, 0, Pi}] == 1/(m^2) && m > 0 && m ∈ Integers, {a, b}, Reals]
```

$$m \in \text{Integers} \ \&\& \ m \geq 1 \ \&\& \ b = \frac{m + a m - a m \cos[m\pi] - a m^2 \pi \sin[m\pi]}{2 m \pi \cos[m\pi] - 2 \sin[m\pi] + m^2 \pi^2 \sin[m\pi]}$$

Следователно, за  $b = \frac{m + a m - a m \cos[m\pi] - a m^2 \pi \sin[m\pi]}{2 m \pi \cos[m\pi] - 2 \sin[m\pi] + m^2 \pi^2 \sin[m\pi]}$  равенството е изпълнено за всяко  $m$  цяло положително.

```
Remove[a, b, m, t]
```

### Задача 28

```
y[x_] := Cos[x] / 2 + (e^x + e^(-x)) / 4
```

```
Integrate[(x - t) y[t], {t, 0, x}]
```

$$\frac{1}{2} (-\cos[x] + \cosh[x])$$

```
Reduce[Cos[x] / 2 + (e^x + e^(-x)) / 4 == Cos[x] + 1/2 (-Cos[x] + Cosh[x]), x]
```

```
True
```

Следователно равенството е изпълнено за всяко  $x \rightarrow y(x)$  е решение на уравнението.

```
Remove[x, y, t]
```

### Задача 29

```
DSolve[{y'[x] - 5 y[x] + 6 y[x] == Sin[2 x] + Cos[x], y'[0] == 0, y[0] == 0}, y[x], x]
```

$$\left\{ \left\{ y[x] \rightarrow \frac{1}{260} (-169 e^{2x} + 118 e^{3x} + 26 \cos[x] + 25 \cos[2x] - 26 \sin[x] - 120 \cos[x] \sin[x] + 65 \sin[2x]) \right\} \right\}$$

**Задача 30**

```
Reduce[Mod[a^2, 10 000] == 2016 && a ≥ 1000 && a ≤ 9999 && a ∈ Integers, a]
```

```
Remove[a]
```