ПЪРВА НАЦИОНАЛНА СТУДЕНТСКА ОЛИМПИАДА
ПО КОМПЮТЪРНА МАТЕМАТИКА
„АКАДЕМИК СТЕФАН ДОДУНЕКОВ"
ТЕХНИЧЕСКИ УНИВЕРСИТЕТ - ГАБРОВО
24-26. X. 2012 г.

Решения на задачите за група B

Задача 1

```
x = 2.3
y = 1.2
```
$\sqrt[3]{(x\verb|^|2) / (x - y) + (x - y) / (x + y)}$

```
2.3

1.2

1.72393
```

```
Remove[x, y]
```

Задача 2

```
Solve[x^2 - 2 x - 11 == 0, x]
```
$\{\{x \to 1 - 2\sqrt{3}\}, \{x \to 1 + 2\sqrt{3}\}\}$

$x1 = 1 - 2\sqrt{3}$
$x2 = 1 + 2\sqrt{3}$
```
N[x2^5 - x1^5]
```
$1 - 2\sqrt{3}$

$1 + 2\sqrt{3}$

1863.69

```
Remove[x, x1, x2]
```

Задача 3

$N\left[\left(-\sqrt{3} + i\right)\verb|^|15 \big/ (1 - i)\verb|^|24\right]$

$3.10862 \times 10^{-15} + 8.\ i$

Задача 4

```
Expand[(x - 2)^9 - (x + 5)^5]
```
$-3637 - 821\,x - 5858\,x^2 + 5126\,x^3 - 4057\,x^4 + 2015\,x^5 - 672\,x^6 + 144\,x^7 - 18\,x^8 + x^9$

```
Remove[x]
```

Задача 5

```
Factor[x^8 - 16]
```
$(-2 + x^2)\ (2 + x^2)\ (2 - 2\,x + x^2)\ (2 + 2\,x + x^2)$

```
Remove[x]
```

Задача 6

```
Solve[32 x^4 - 128 x^3 + 114 x^2 + 63 x - 81 == 0, x]
```

$\{\{x \to -\frac{3}{4}\}, \{x \to 1\}, \{x \to \frac{3}{2}\}, \{x \to \frac{9}{4}\}\}$

```
Remove[x]
```

Задача 7

```
Reduce[√(5 - 2 x) < 6 x - 1, x]
```

$\frac{1}{2} < x \leq \frac{5}{2}$

```
Remove[x]
```

Задача 8

```
Reduce[x^2 - x - 2 >= 0, x]
```

$x \leq -1 \,||\, x \geq 2$

Първи случай: e^x <= -1 е невъзможен
Втори случай:

```
Reduce[e^x >= 2, x, Reals]
```

$x \geq \text{Log}[2]$

```
Remove[x]
```

Задача 9

```
Solve[{x + 2 y - z == -3, 2 x + 3 y + z == -1, x - y - z == 3}, {x, y, z}]
```

$\{\{x \to 2, y \to -2, z \to 1\}\}$

```
Remove[x, y, z]
```

Задача 10

```
Det[{{1 + a, 1, 1, 1}, {1, 1 - a, 1, 1}, {1, 1, 1 + b, 1}, {1, 1, 1, 1 - b}}]
```

$a^2 b^2$

```
Remove[a, b]
```

Задача 11

```
Solve[Det[{{x, 1, 2}, {3, x, 4}, {5, 6, x}}] == 20, x]
```

$\{\{x \to 1\}, \{x \to \frac{1}{2}(-1 - \sqrt{145})\}, \{x \to \frac{1}{2}(-1 + \sqrt{145})\}\}$

```
Remove[x]
```

Задача 12

```
Solve[{{1, 2}, {3, 4}}.{{a, 6}, {c, 7}} == {{a, 6}, {c, 7}}.{{1, 2}, {3, 4}}, {a, c}]
```

$\{\{a \to -2, c \to 9\}\}$

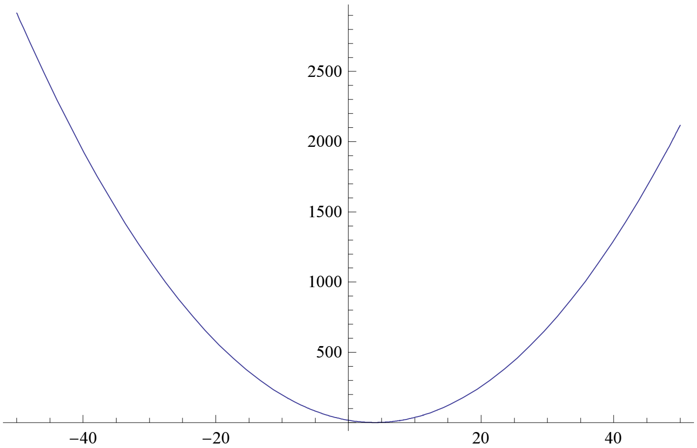
```
Remove[a, c]
```

Задача 13

```
Solve[{4 + 2 b + c == 4, 25 + 5 b + c == 1}, {b, c}]
```

$\{\{b \to -8, c \to 16\}\}$

```
Plot[x^2 - 8 x + 16, {x, -50, 50}]
```




```
Remove[x, b, c]
```

Задача 14

```
Limit[ArcSin[(1 - x^2) / (1 + x^2)], x → ∞]
```

$-\frac{\pi}{2}$

```
Remove[x]
```

Задача 15

```
D[e^√x Sin[x]^2, x]
```

$2\, e^{\sqrt{x}}\, \mathrm{Cos}[x]\, \mathrm{Sin}[x] + \frac{e^{\sqrt{x}}\, \mathrm{Sin}[x]^2}{2 \sqrt{x}}$

```
Remove[x]
```

Задача 16

```
f[x_] := ArcCos[√x]
f''[1 / 4]
```

$\frac{8}{3 \sqrt{3}}$

```
Remove[x, f]
```

Задача 17

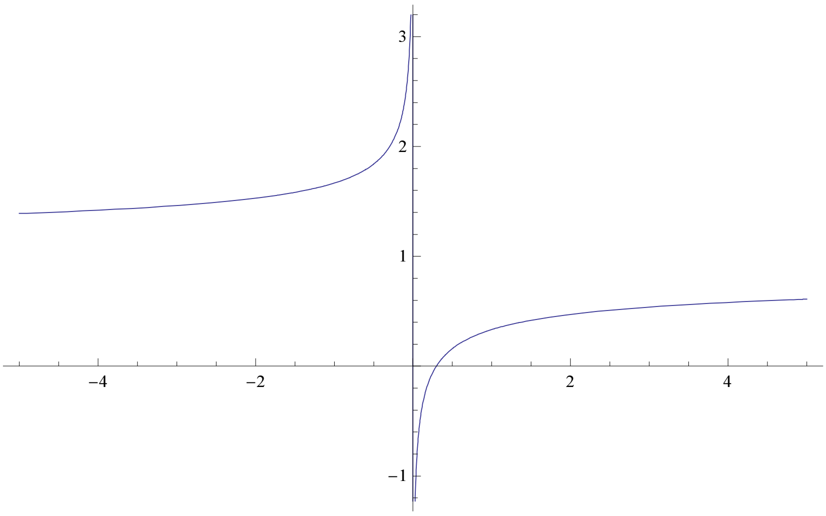
```
f = x - ∛x^2
```

$x - (x^2)^{1/3}$

```
g = D[f, x]
```

$$1 - \frac{2\,x}{3\,(x^2)^{2/3}}$$

```
Plot[g, {x, -5, 5}]
```




```
Solve[g == 0, x]
```

$$\left\{\left\{x \to \frac{8}{27}\right\}\right\}$$

Следователно f(x) е растяща в интервала (-∞; 0)⋃(8/27; +∞) и намаляваща в интервала (0; 8/27).

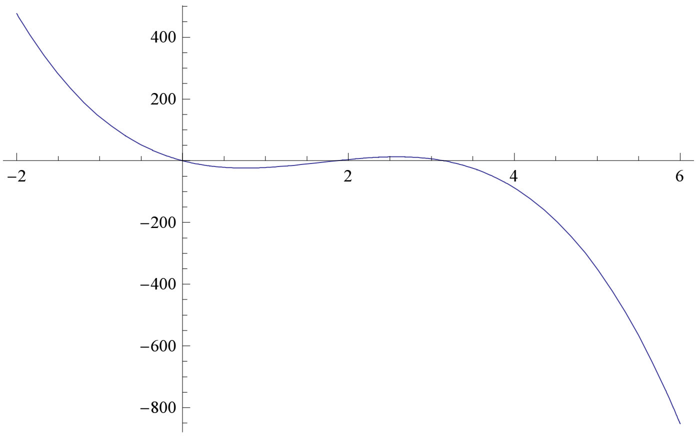
```
Remove[f, g, x]
```

Задача 18

Разглеждаме производната:

```
df = D[-3 x^4 + 20 x^3 - 35 x^2 + 24, x]
Plot[df, {x, -2, 6}]
Solve[df == 0, x]
```

$$-70\,x + 60\,x^2 - 12\,x^3$$



$$\left\{\{x \to 0\},\ \left\{x \to \frac{1}{6}\left(15 - \sqrt{15}\right)\right\},\ \left\{x \to \frac{1}{6}\left(15 + \sqrt{15}\right)\right\}\right\}$$

```
f[x_] = -3 x^4 + 20 x^3 - 35 x^2 + 24
```

$24 - 35 x^2 + 20 x^3 - 3 x^4$

Локални максимуми:

```
f[0]
```

$N\left[f\left[\frac{1}{6}\left(15+\sqrt{15}\right)\right]\right]$

24

6.46248

Локален Минимум:

$N\left[f\left[\frac{1}{6}\left(15-\sqrt{15}\right)\right]\right]$

-4.29581

```
Remove[x, f, df]
```

Задача 19

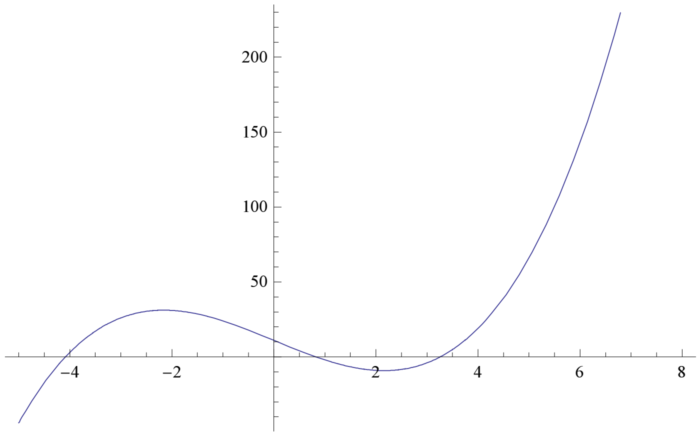
```
FindMaximum[{(Log[x] - 1) / x, x > 0}, x]
```

{0.135335, {x → 7.38868}}

```
Remove[x]
```

Задача 20

```
Plot[x^3 - 14 x + 11, {x, -5, 8}]
```

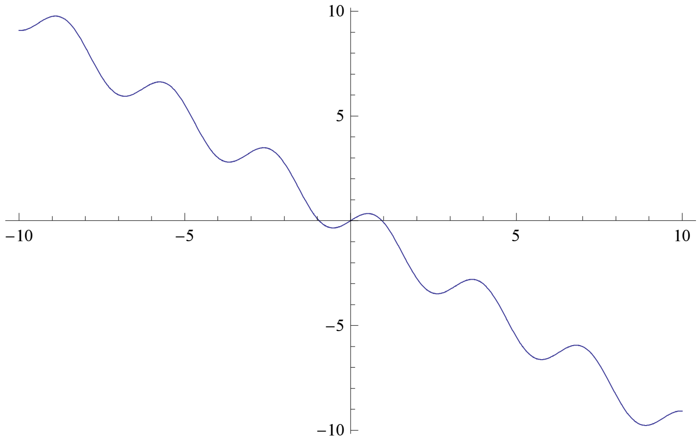


Положителните корени са два.

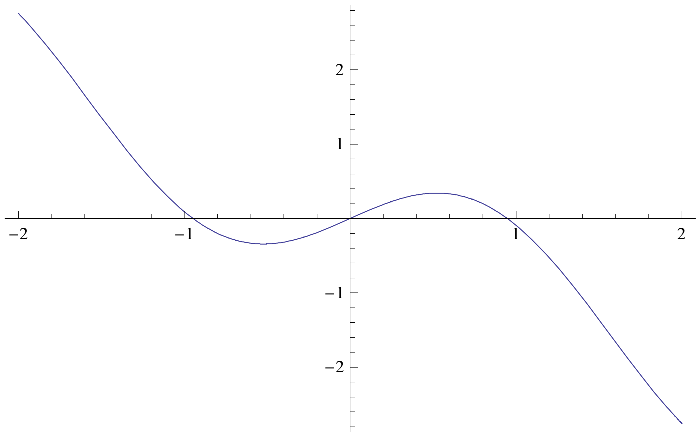
```
Remove[x]
```

Задача 21

```
Plot[Sin[2 x] - x, {x, -10, 10}]
```



```
Plot[Sin[2 x] - x, {x, -2, 2}]
```



```
FindRoot[Sin[2 x] - x, {x, -1}]
```

{x → - 0.947747}

```
FindRoot[Sin[2 x] - x, {x, 1}]
```

{x → 0.947747}

```
Remove[x]
```

Задача 22

```
Solve[(x^2 - x + 1) / (x^2 + x + 1) == a, x, Reals]
```

$$\left\{\left\{x \to \text{ConditionalExpression}\left[\frac{-1-a}{2\,(-1+a)} - \frac{1}{2}\sqrt{\frac{-3+10\,a-3\,a^2}{(-1+a)^2}},\ \frac{1}{3} < a < 3\right]\right\},\right.$$

$$\left.\left\{x \to \text{ConditionalExpression}\left[\frac{-1-a}{2\,(-1+a)} + \frac{1}{2}\sqrt{\frac{-3+10\,a-3\,a^2}{(-1+a)^2}},\ \frac{1}{3} < a < 3\right]\right\}\right\}$$

Следователно за a∈(1/3; 3) уравнението има решение в областта на реалните числа.

```
Remove[x]
```

Задача 23

```
Integrate[1/((x+1) Sqrt[1-x^2]), x]
```

$$-\frac{-1+x}{\sqrt{1-x^2}}$$

```
Remove[x]
```

Задача 24

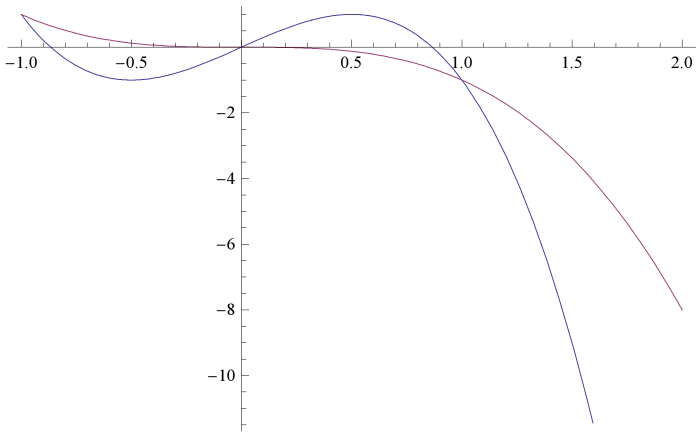
```
Integrate[Sqrt[x^2-4]/x^4, {x, 2, 4}]
```

$$\frac{\sqrt{3}}{32}$$

```
Remove[x]
```

Задача 25

```
Plot[{3 x - 4 x^3, -x^3}, {x, -1, 2}]
```




```
Solve[3 x - 4 x^3 == -x^3, x]
```

{{x → -1}, {x → 0}, {x → 1}}

```
Integrate[3 x - 4 x^3 + x^3, {x, 0, 1}]
```

$$\frac{3}{4}$$

```
Remove[x]
```

Задача 26

$$\sum_{k=1}^{19} k^2/(k+1)^2$$

$$\frac{156\,081\,455\,609\,383\,441}{10\,838\,475\,198\,270\,720}$$

```
Remove[k]
```

Задача 27

```
Solve[Sum[1/(k*(k+2)), {k, 1, n}] == 3038623/4054182 && n ≥ 0, n, Integers]
```

$$\text{Solve}\left[\sum_{k=1}^{n}\frac{1}{k*(k+2)} == \frac{3\,038\,623}{4\,054\,182}\ \&\&\ n \geq 0,\ n,\ \text{Integers}\right]$$

{{n → 2012}}

```
Remove[n, k]
```

Задача 28

$$\text{Limit}\left[\sum_{n=1}^{\infty} 1/n\wedge 4,\ n \to \infty\right]$$

$$\frac{\pi^4}{90}$$

```
Remove[n]
```

Задача 29

```
y[x_] := e^x - 1
```

$$\text{Integrate}\left[e^{2\,(x-t)}\ y'[t],\ \{t,\ 0,\ x\}\right]$$

$e^x\ (-1 + e^x)$

$$\text{Reduce}\left[y''[x] + e^x\ (-1 + e^x) == e^{2x},\ x\right]$$

True

Изразът е тъждество, следователно функцията е решение на уравнението.

```
Remove[x, y]
```

Задача 30

```
DSolve[{y''[x] - 3 y'[x] + 2 y[x] == 0, y'[0] == 2, y[0] == 3}, y[x], x]
```

$\{\{y[x] \to -e^x\ (-4 + e^x)\}\}$